Lecture 12: Language models

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Outline

- What is a language model?
- Applications of language models
- N-gram and chain rule
  - Examples for bigram probabilities
- Evaluating language models
- Smoothing

Give a word

The student is watching ________

Probabilistic language model

- Goal: Compute the probability of a sentence or sequence of words

$$P(W) = P(w_1, w_2, w_3, \ldots, w_n)$$

- Probability of an upcoming word

$$P(w_n \mid w_1, w_2, w_3, \ldots, w_{n-1})$$

LM applications

- Machine translation

$$P(\text{Students from my class are the best} \mid \text{我班上的学生是最棒的})$$
$$> P(\text{Students from Stanford are the best} \mid \text{我班上的学生是最棒的})$$

- Natural language generation

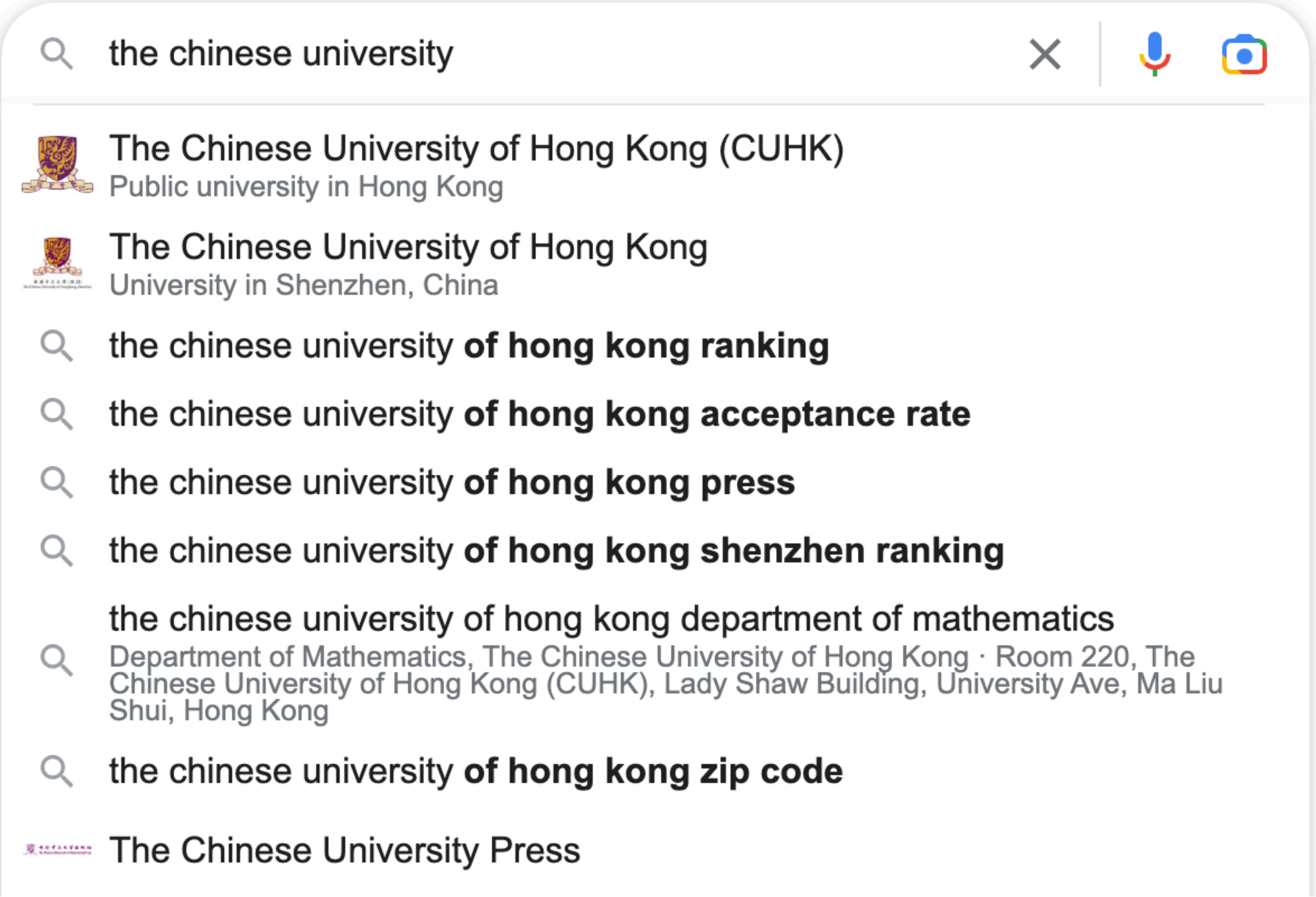
$$P(\text{best} \mid \text{Students from my class are the}) > P(\text{average} \mid \text{Students from my class are the})$$

- Speech recognition

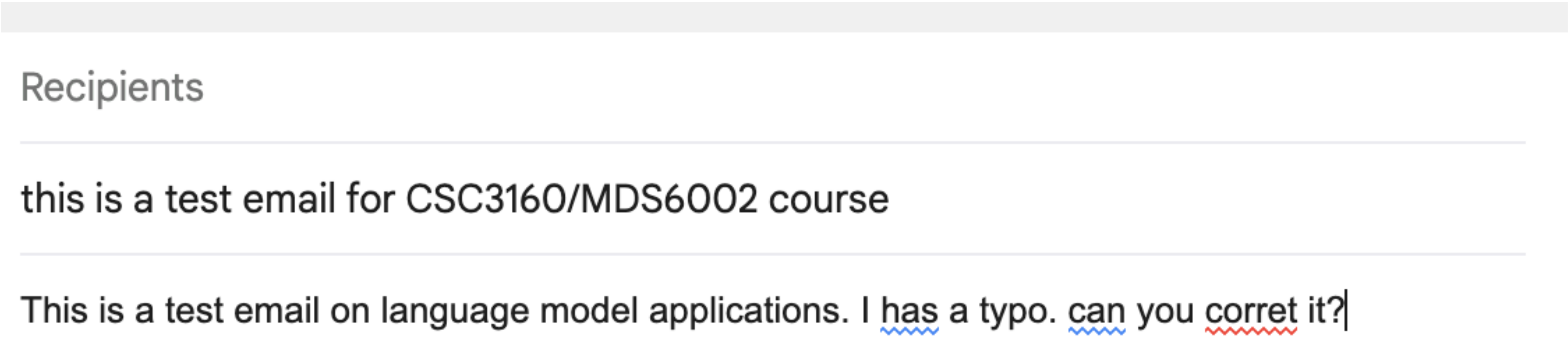
$$P(\text{Three students}) > P(\text{Tree students})$$

Language models in daily life

Google



Language models in daily life



Probability of next word

$$P(\text{best} \mid \text{Students from my class are the}) = \frac{C(\text{Students from my class are the best})}{C(\text{Students from my class are the})}$$

- C(Students from my class are the best) is count of the phrase "*Students from my class are the best*"

Probability of next word

- Smarter way to estimate the probability

$$P(\text{Students from my class are the best})$$

$$= P(\text{best} \mid \text{the})P(\text{the} \mid \text{are})P(\text{are} \mid \text{class})P(\text{class} \mid \text{my})P(\text{my} \mid \text{from})P(\text{from} \mid \text{Students})P(\text{Students})$$

- Chain rule of probability

$$P(w_{1:n}) = P(w_1)P(w_2 \mid w_1)P(w_3 \mid w_{1:2}) \ldots P(w_n \mid w_{1:n-1})$$

N-gram

The student is watching______

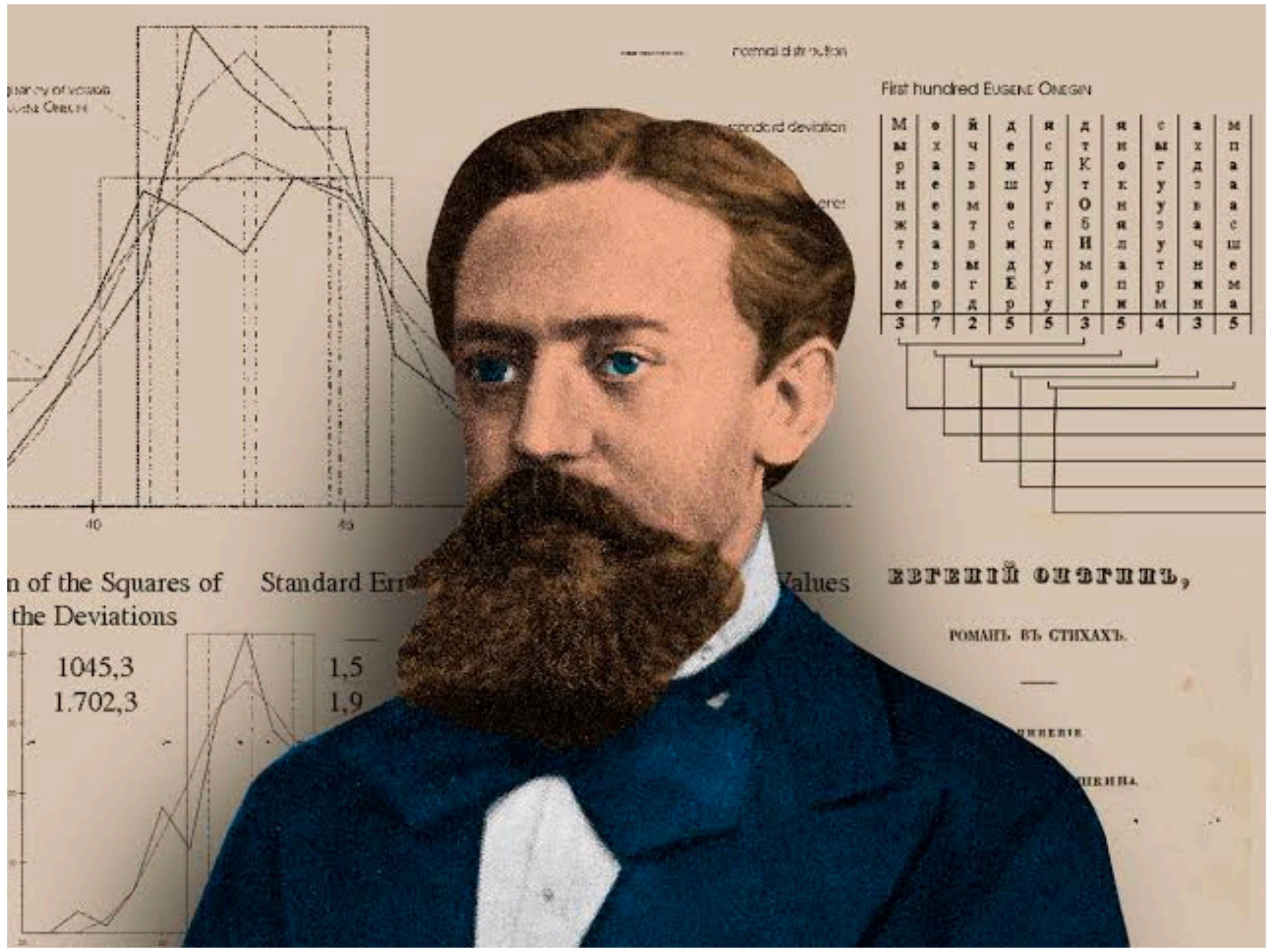
Unigram: “The”
Bigram: “The student”
Trigram: “The student is”
4-gram: “The student is watching”

Bigram model

- approximates the probability of a word given **all the previous words** by using only the conditional probability of the **preceding word**

$$P(\text{best} \mid \text{Students from my class are the}) \approx P(\text{best} \mid \text{the})$$

Markov assumption



- Assumption: the probability of a word depends only on the previous word

$$P(\text{best} \mid \text{Students from my class are the}) \approx P(\text{best} \mid \text{the})$$

$$P(w_n \mid w_{1:n-1}) \approx P(w_n \mid w_{n-1})$$

- Markov model: assume we can predict the probability of some future unit without looking too far into the past

Generalizing bigram to n-gram

- From bigram to n-gram

$$P(w_n \mid w_{1:n-1}) \approx P(w_n \mid w_{n-N+1:n-1})$$

- N = 2: bigram
- N = 3: trigram
- N = 4: 4-gram
- N = 5: 5-gram

Simplest case: unigram

$$P(w_{1:n}) = P(w_1)P(w_2)P(w_3)\ldots P(w_n)$$

regenerate by simulating an unigram model

Certainly, I can simulate an unigram model to generate a new sentence. Here is an example:

"Dog cat ball blue happy tree sing boy laugh jump sun"

Bigram model

- Condition on the previous word

$$P(w_i \mid w_{1:i-1}) \approx P(w_i \mid w_{i-1})$$

regenerate by simulating a bigram model

Sure, here's an example sentence generated using a bigram model:

"The sun is shining brightly, and the birds are singing in the trees."

Example with a mini-corpus

```
<s> I am Sam </s>
<s> Sam I am </s>
<s> I do not like green eggs and ham </s>
```

<s> : beginning symbol

</s>: ending symbol

- Maximum-likelihood estimation (MLE): bigram probability

$$P(\texttt{I}|\texttt{<s>}) = \frac{2}{3} = .67 \qquad P(\texttt{Sam}|\texttt{<s>}) = \frac{1}{3} = .33 \qquad P(\texttt{am}|\texttt{I}) = \frac{2}{3} = .67$$

$$P(\texttt{</s>}|\texttt{Sam}) = \frac{1}{2} = 0.5 \qquad P(\texttt{Sam}|\texttt{am}) = \frac{1}{2} = .5 \qquad P(\texttt{do}|\texttt{I}) = \frac{1}{3} = .33$$

$$P(w_n|w_{n-N+1:n-1}) = \frac{C(w_{n-N+1:n-1}\, w_n)}{C(w_{n-N+1:n-1})}$$

A slightly large example

- Bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

- Unigram counts

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

  - “I want” occurred 827 times in the document.
  - “want want” occurred 0 times.

Bigram probabilities

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

- Other useful probabilities

$$P(\texttt{i}|\texttt{<s>}) = 0.25 \qquad P(\texttt{english}|\texttt{want}) = 0.0011$$
$$P(\texttt{food}|\texttt{english}) = 0.5 \qquad P(\texttt{</s>}|\texttt{food}) = 0.68$$

- Calculate probability of sentences like "*I want English food*"

$$\begin{aligned} &P(\texttt{<s> i want english food </s>}) \\ &\quad = P(\texttt{i}|\texttt{<s>})P(\texttt{want}|\texttt{i})P(\texttt{english}|\texttt{want}) \\ &\qquad\quad P(\texttt{food}|\texttt{english})P(\texttt{</s>}|\texttt{food}) \\ &\quad = .25 \times .33 \times .0011 \times 0.5 \times 0.68 \\ &\quad = .000031 \end{aligned}$$

Evaluating language models

Perplexity

- the inverse probability of the test set, normalized by the number of words

$$\begin{aligned}\text{perplexity}(W) &= P(w_1 w_2 \ldots w_N)^{-\frac{1}{N}} \\ &= \sqrt[N]{\frac{1}{P(w_1 w_2 \ldots w_N)}}\end{aligned}$$

- Applying chain rule

$$\text{perplexity}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i|w_1 \ldots w_{i-1})}}$$

Intuition of perplexity

- Intuitively, perplexity can be understood as a measure of uncertainty

- What's the level of uncertainty to predict the next word?
  - The current president of CUHK Shenzhen is ________ ?
  - ChatGPT is built on top of OpenAI's GPT-3 family of large language ______ ?

- Uncertainty level
  - Unigram: highest
  - Bigram: high
  - 5-gram: low

Lower perplexity = better model

	Unigram	Bigram	Trigram
Perplexity	962	170	109

Model	PPL
Trigram-1	303.2
Trigram-all	112.2
5gram-1	281.0
5-gram-all	73.7
ME-1	286.5
ME-all	68.8
FFNN-all	83.0
RNN-1	211.1
RNN-all	45.7
RNNME-1	196.3
RNNME-3	136.0
RNNME-6	109.7
RNNME-9	107.5
RNNME-12	103.1
RNNME-15	91.3
RNNME-18	106.9
RNNME-21	78.9
L-1-512-512-0.1	63.2
L-1-1024-512-0.1	54.5
L-1-2048-512-0.1	45.3
L-1-8192-2048-0.5	35.9
L-1-8192-2048-0	37.5
L-2-2048-512-0.1	39.8
L-2-4096-1024-0.1	33.6
Human (estimated)	12.0

https://web.stanford.edu/~jurafsky/slp3/3.pdf

https://www.isca-speech.org/archive_v0/Interspeech_2017/pdfs/0729.PDF

Long tail


TO KOYTOYKAKI
TAVERNA
1901
COFFE
BAR
CASTELLO
PIZZA
ΠΠΛΕΙΤΑΙ
Λαϊκο καφωδειο.
ΤΑΒΕΡΝΑ
ο ΚΑΝΑΔΟΣ
CASTLE
Imaret
Hotel

The perils of overfitting

- N-gram models only work well for word prediction if the test corpus looks like the training corpus
  - In real world, the inference corpus often doesn't look like the training
  - Robust models that generalize are all we need
  - One kind of generalization: **Zeros**
    - Things that doesn't ever occur in the training set but not in the test set

Zeros

- Training set
  - … denied the allegations
  - … denied the reports
  - … denied the claims
  - … denied the request
- Test set
  - … denied the offer
  - … denied the loan

$$P(\text{offer} \mid \text{denied the}) = 0$$

$$P(\text{loan} \mid \text{denied the}) = 0$$

Zero probability bigrams

- Bigram with zero probability
  - On test set $$P(w_i \mid w_{1:i-1}) \approx P(w_i \mid w_{i-1})$$

- Perplexity: can't compute because of 1 over 0…

$$\text{perplexity}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i|w_1 \ldots w_{i-1})}}$$

Unseen events

Training data: The wolf is an endangered species
Test data: The wallaby is endangered

Unigram	Bigram	Trigram
P(the)	P(the | <s>)	P(the | <s>)
× P(wallaby)	× P(wallaby | the)	× P(wallaby | the, <s>)
× P(is)	× P(is | wallaby)	× P(is | wallaby, the)
× P(endangered)	× P(endangered | is)	× P(endangered | is, wallaby)

- **Case 1:** P(wallaby), P(wallaby | the), P(wallaby | the, <s>):
  What is the probability of an unknown word (in any context)?
- **Case 2:** P(endangered | is)
  What is the probability of a known word in a known context, if that word hasn't been seen in that context?
- **Case 3:** P(is | wallaby) P(is | wallaby, the) P(endangered | is, wallaby):
  What is the probability of a known word in an unseen context?

https://courses.engr.illinois.edu/cs447/fa2018/Slides/Lecture04.pdf

What can we do?

Dealing with unknown words: Simple solution

- Create an unknown word token <UNK>
  - Training of <UNK> probabilities
  - Create a fixed lexicon **L** of size **V**
  - At text normalization phase, any training word not in L changed to <UNK>

- During inference
  - Use UNK probabilities for any word not in training

Smoothing

- To improve the accuracy of our model
- To handle data sparsity, out of vocabulary words, words that are absent in the training set.
- Smoothing techniques
  - Laplace smoothing: Also known as add-1 smoothing
  - Additive smoothing
  - Good-turing smoothing
  - Kneser-Ney smoothing
  - Katz smoothing
  - Church and Gale Smoothing

Laplace Smoothing

- Assuming every (seen or unseen) event occurred once more than it did in the training data.

$$P_{\text{Laplace}}(w_n \mid w_{n-1}) = \frac{C(w_{n-1}, w_n) + 1}{C(w_{n-1}) + V}$$

Bigram counts

Original

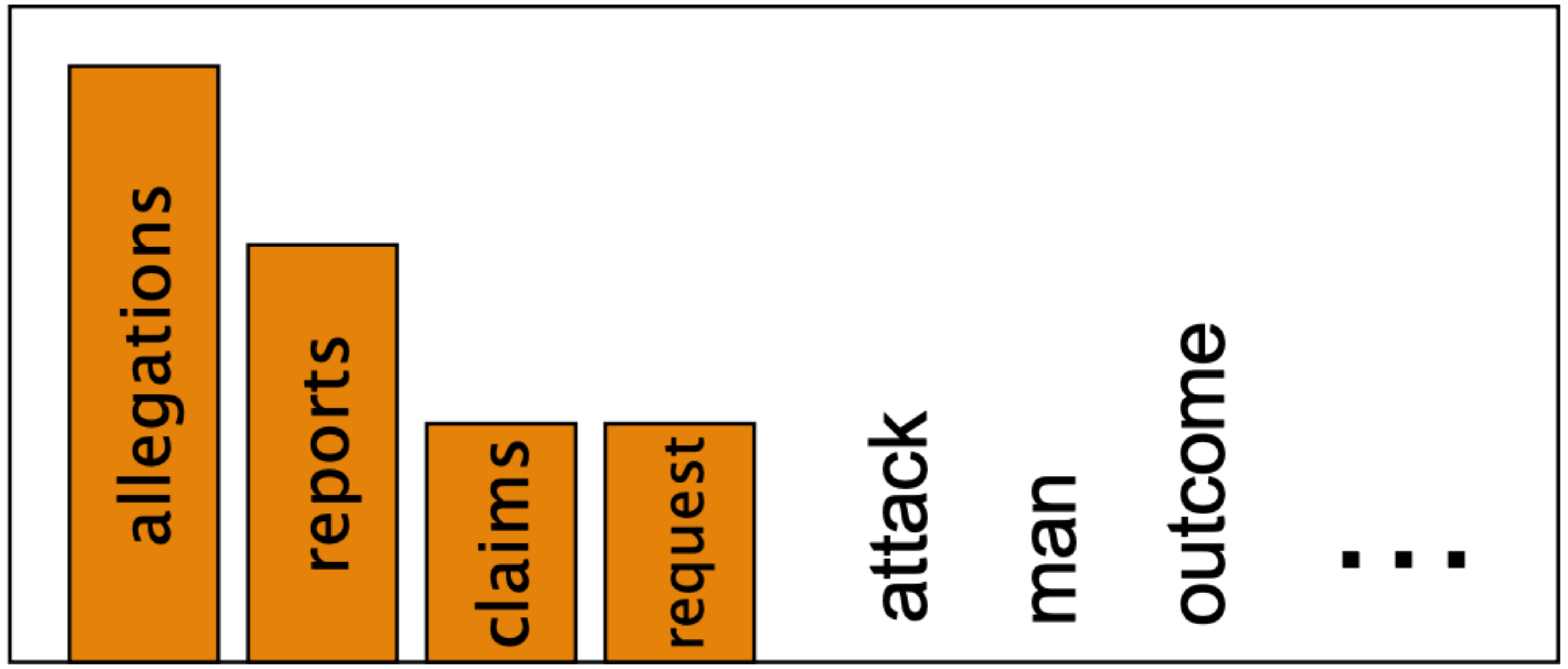
	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

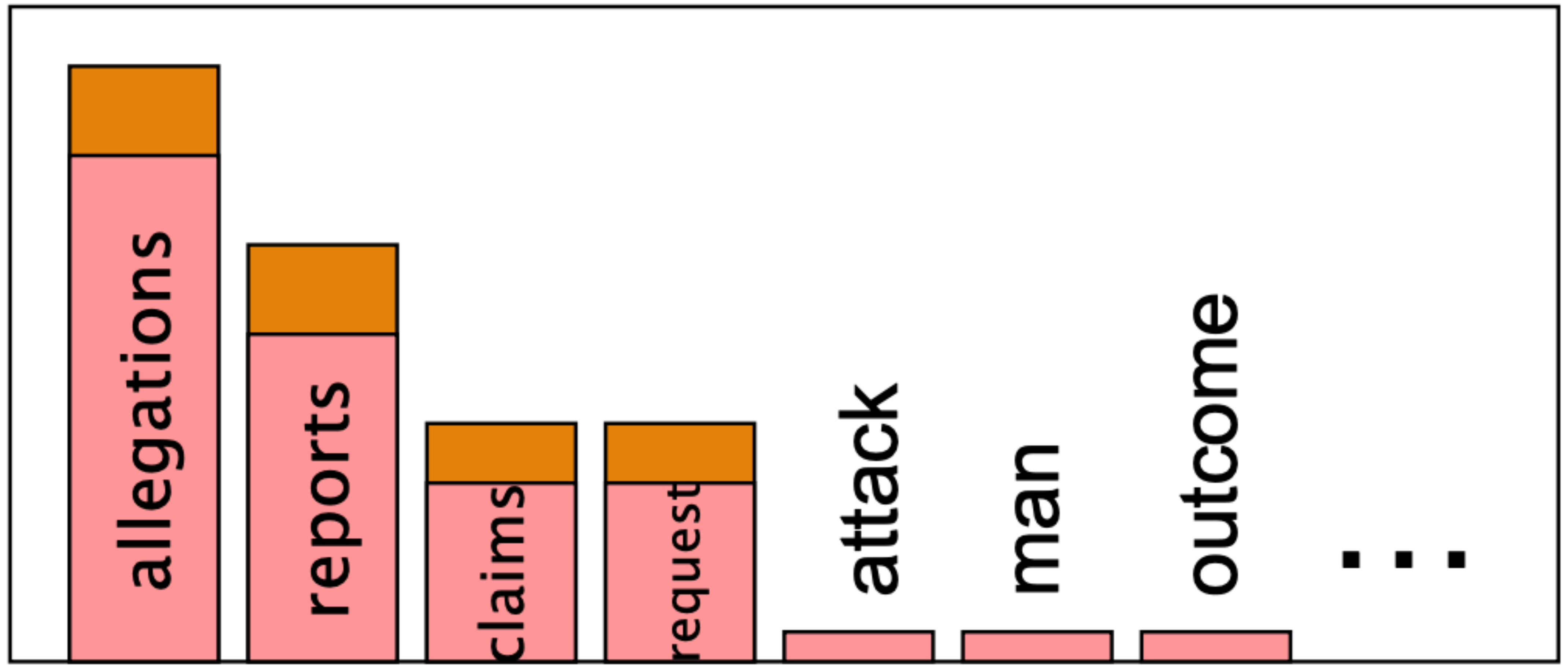
Smoothed

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Intuition of smoothing

- When we have sparse statistics:
  - P(w I denied the)
    - 3 allegations
    - 2 reports
    - 1 claims
    - 1 request
- Steal probability mass to generalize better
  - P(w I denied the)
    - 2.5 allegations
    - 1.5 reports
    - 0.5 claims
    - 0.5 request
    - 2 other





Backoff and interpolation

- Use less context
  - Backoff
    - use trigram if you have good evidence,
    - otherwise bigram, otherwise unigram
  - Interpolation
    - Mix unigram, bigram, trigram

Summary

- Language model
  - Compute the probability of a sentence or sequence of words
  - Predicting next word
- N-gram
  - Unigram
  - Bigram
  - Trigram
  - Etc
- Evaluating language model: perplexity
- Smoothing

Reading

- Chapter 3: N-gram Language Models
  - https://web.stanford.edu/~jurafsky/slp3/3.pdf